V E R I F I C A T I O N # 1

Projectile

The purpose of this verification is to confirm that the projectile algorithm used by the program is working correctly. The projectile algorithm calculates the motion of the rocks while they are travelling through the air, bouncing from one point on the slope to another. The vast majority of the simulation time in RocFall takes place in the projectile algorithm. Any errors in the projectile algorithm would surely produce incorrect results. Therefore, it is essential that the projectile algorithm work correctly.

The example consists of a slope with two benches and a single rock that begins its travel at the crest of the slope. The rock was given an initial velocity and bounced a number of times before coming to rest at the base of the slope. The initial velocity for the rock was chosen so that the rock would follow a distinct path (high and clearly above the slope) and so that the rock would have enough energy so that each of the impacts would occur on a different segment of the slope. This velocity does not necessarily reflect typical initial velocities that are used in rockfall analyses.

The slope was created by making minor modifications to the geometry of an actual slope profile. The geometry was modified so that the impacts would occur on slope segments with a positive slope, a negative slope and a horizontal segment. This was done in order to verify that the projectile algorithm handles sign changes correctly. This verification also serves as a good example of the sign conventions that are used in the program.

The slope geometry and the input parameters were configured so that no sliding would occur. No statistics were incorporated into this verification (i.e. only mean values were used, all standard deviations were set to 0). Although rock trajectories in an actual simulation typically have dozens of steps, only the first four steps are followed here. This was done in the interest of brevity.

The minimum velocity (V_{MIN}) was set to 1 m/s. This minimum velocity was selected so that the simulation did not end before the four steps were complete. Other numbers used in this example (e.g. the mass of the rock) were selected primarily for their ease in manual calculations.

Initial Conditions

The rock starts at location X_0 = 0 m, Y_0 = 60 m (which coincides with the first slope vertex). The rock was given an initial velocity of V_{X0} = 7 m/s, V_{Y0} = 2 m/s and a mass of 10 Kg.

The location of the slope vertices and the coefficients of restitution for each slope segment are presented in the following table:

	x co-ordinate	y co-ordinate	R_N	R_T
Vertex 1	0	60		
Segment 1			0.5	0.8
Vertex 2	7	39		
Segment 2			0.5	0.8
Vertex 3	19	40		
Segment 3			0.5	0.8
Vertex 4	26	22		
Segment 4			0.6	0.9
Vertex 5	38	20		
Segment 5			0.6	0.9
Vertex 6	46	0		
Segment 6			0.4	0.6
Vertex 7	89	0		

Table A.1.1 - Slope geometry

A diagram of the rock trajectory in RocFall and the comparison program (rockfall) follow (Figures A.1.1 and A.1.2):

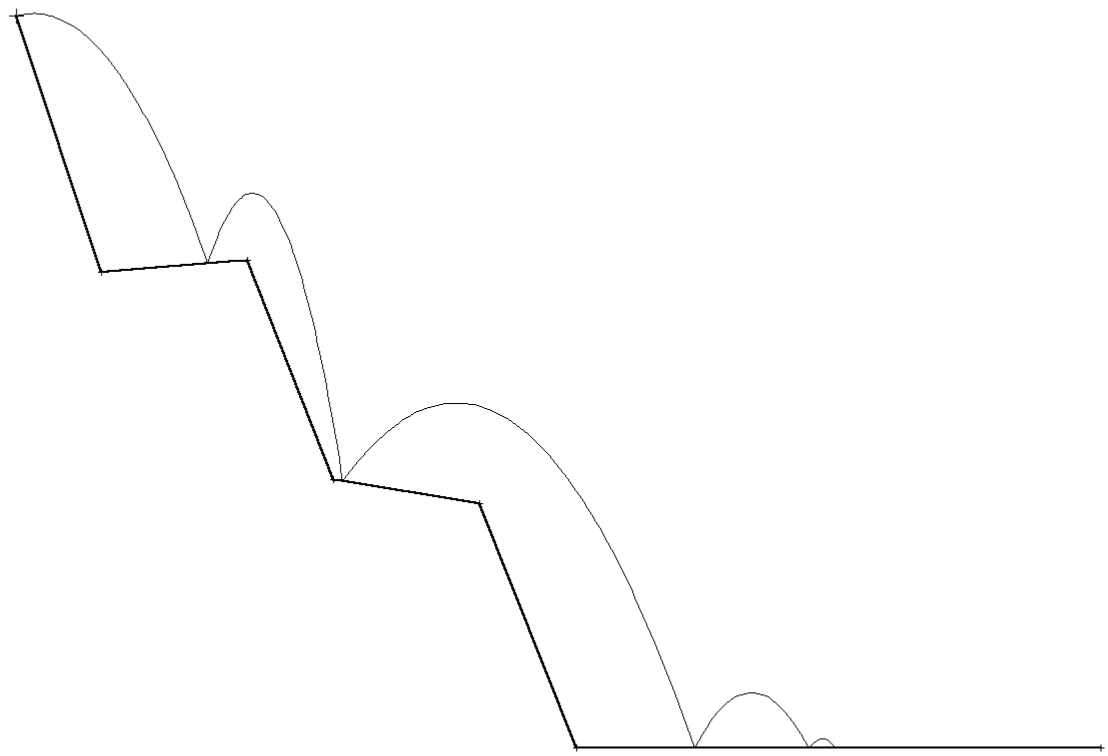

Figure A.1.1 - Rock trajectory in RocFall

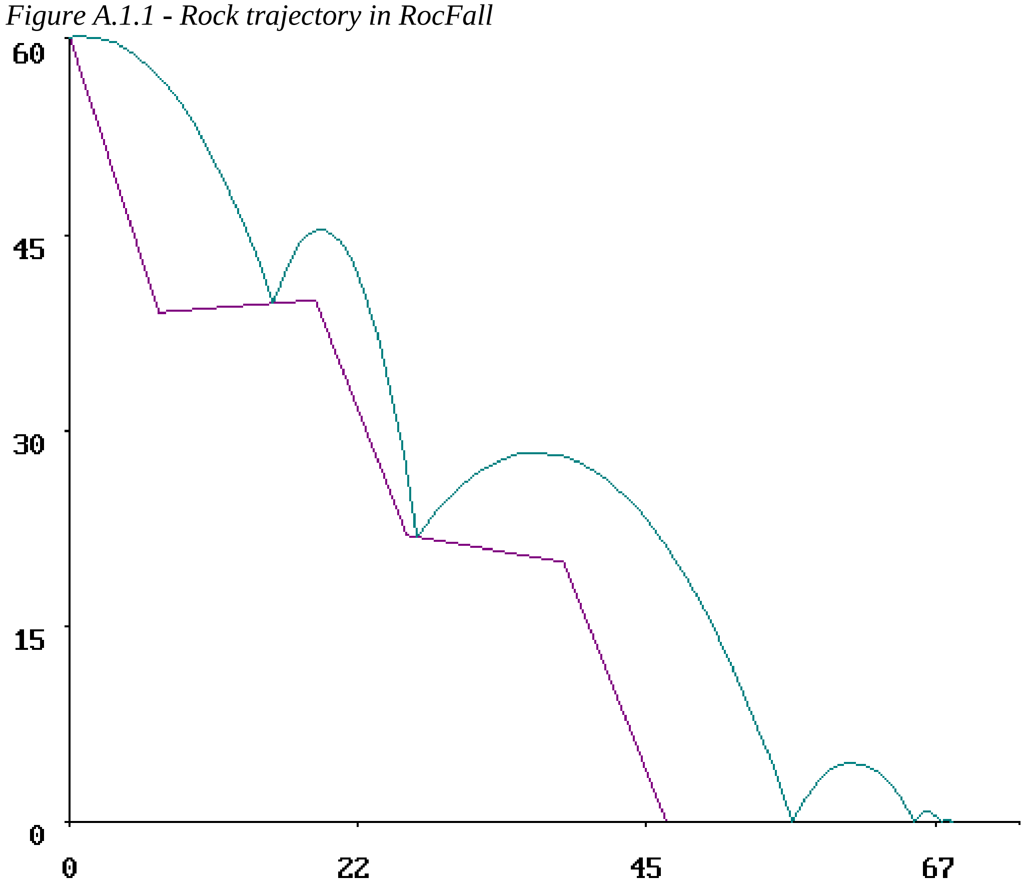

Figure A.1.2 - Rock trajectory in comparison program

Hand Calculations

The projectile algorithm consists, mainly, of the process of determining the intersection between a parabola (the path the rock follows while it is in the air) and a line segment (one of the slope segments). The location of the parabola-line intersection is the roots of the quadratic equation (equation 4.7):

$$\left[\frac{1}{2}g\right]t^2 + \left[V_{Y0} - qV_{X0}\right]t + \left[Y_0 - Y_1 + q(X_1 - X_0)\right] = 0$$

Each step consists of determining the necessary parameters and solving the quadratic equation to find the intersection point. Once the intersection point is found, the impact is calculated. If the rock has enough velocity after the impact, as determined by a comparison to the minimum velocity (V_{MIN}), another step is initiated.

In the interest of brevity, the process of searching for the slope segment where the impact occurs has been left out of the verification.

Step 1

The rock starts at location X_0 = 0 m, Y_0 = 60 m (which coincides with the first slope vertex). The rock was given an initial velocity of V_{X0} = 7 m/s, V_{Y0} = 2 m/s. The necessary parameters are determined and the quadratic equation is solved to find the time of intersection with the second slope segment (using equations 4.8, 4.9, 4.10, 4.11 and 4.12):

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(40-39)}{(19-7)} \cong 0.0833$$

$$a = \frac{1}{2}g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = 2 - (0.0833)7 \cong 1.417$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 60 - 39 + 0.0833(7-0) \cong 21.58$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(1.417) \pm \sqrt{(1.417)^2 - 4(-4.90)(21.58)}}{2(-4.90)} \cong -1.959 \ or \ 2.25 \ s$$

t = -1.959 s is rejected because t must lie in the range [0,∞]. The intersection point and pre-intersection velocity are found by substituting t back into equations 4.3, 4.4, 4.5 and 4.6:

$$X_I = V_{X0}t + X_0 = 7(2.25) + 0 = 15.732\ m$$

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(2.25)^2 + 2(2.25) + 60 = 39.728\ m$$

$$V_{XB} = V_{X0} = 7\ m/s$$

$$V_{YB} = V_{Y0} + gt = 2 + (-9.81)2.25 = -20.0\ m/s$$

The velocities are transformed into components normal and tangential to the slope segment (using equations 4.13 and 4.14):

$$\theta = \tan^{-1}(q) = 4.77^0$$

$$V_{NB} = (V_{YB})\cos(\theta) - (V_{XB})\sin(\theta) = (-20.0)\cos(4.77) - (7)\sin(4.77) = -20.5\ m/s$$

$$V_{TB} = (V_{YB})\sin(\theta) + (V_{XB})\cos(\theta) = (-20.0)\sin(4.77) + (7)\cos(4.77) = 5.31\ m/s$$

The impact is calculated by multiplying by the coefficients of restitution (using equations 4.15 and 4.16):

$$V_{NA} = R_N V_{NB} = 0.5(-20.5) = -10.28\ m/s$$

$$V_{TA} = R_T V_{TB} = 0.8(5.31) = 4.25\ m/s$$

The velocities are transformed back into vertical and horizontal components (using equations 4.17 and 4.18):

$$V_{XA} = (V_{NA})\sin(\theta) + (V_{TA})\cos(\theta) = (-10.28)\sin(4.77) + (4.25)\cos(4.77) = 3.38\ m/s$$

$$V_{YA} = (V_{TA})\sin(\theta) - (V_{NA})\cos(\theta) = (4.25)\sin(4.77) - (-10.28)\cos(4.77) = 10.59\ m/s$$

Step 1 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(3.38)^2 + (10.59)^2} = 11.12\ m/s$$

Since the velocity of the rock, V_{CHECK} (= 11.12 m/s) is greater than the minimum velocity, V_{MIN} (= 1.0 m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step.

Step 2

The final rock conditions for step 1 are used as the initial conditions for step 2. That is:

$$X_{0(step2)} = X_{I(step1)}$$

$$Y_{0(step2)} = Y_{I(step1)}$$

$$V_{X0(step2)} = V_{XA(step1)}$$

$$V_{Y0(step2)} = V_{YA(step1)}$$

The necessary parameters are determined and the quadratic equation is solved to find the time to intersection with the fourth slope segment:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(20-22)}{(38-26)} \cong -0.1667$$

$$a = \frac{1}{2} g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = 10.59 - (-0.1667)3.88 \cong 11.16$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 39.73 - 22 + (-0.1667)(26 - 15.7) \cong 16.02$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(11.16) \pm \sqrt{(11.16)^2 - 4(-4.90)(16.02)}}{2(-4.90)} \cong -0.998 \; or \; 3.27 \; s$$

t = -0.998 s is rejected because *t* must lie in the range [0,∞]. The intersection point and pre-impact velocity are determined:

$$X_I = V_{X0}t + X_0 = 3.88(3.27) + 15.73 = 26.800 \; m$$

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(3.27)^2 + 11.16(3.27) + 39.7 = 21.867 \; m$$

$$V_{XB} = V_{X0} = 3.88 \; m/s$$

$$V_{YB} = V_{Y0} + gt = 11.16 + (-9.81)3.27 = -21.5\ m/s$$

The velocities are transformed into components normal and tangential to the slope segment:

$$\theta = \tan^{-1}(q) = -9.46^0$$

$$V_{NB} = (V_{YB})\cos(\theta) - (V_{XB})\sin(\theta) = (-21.5)\cos(-9.46) - (3.38)\sin(-9.46) = -20.6\ m/s$$

$$V_{TB} = (V_{YB})\sin(\theta) + (V_{XB})\cos(\theta) = (-21.5)\sin(-9.46) + (3.38)\cos(-9.46) = 6.87\ m/s$$

The impact is calculated by multiplying by the coefficients of restitution:

$$V_{NA} = R_N V_{NB} = 0.6(-20.6) = -12.4\ m/s$$

$$V_{TA} = R_T V_{TB} = 0.8(6.87) = 6.18\ m/s$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = (V_{NA})\sin(\theta) + (V_{TA})\cos(\theta) = (-12.4)\sin(9.46) + (6.18)\cos(9.46) = 8.14\ m/s$$

$$V_{YA} = (V_{TA})\sin(\theta) - (V_{NA})\cos(\theta) = (6.18)\sin(9.46) - (-12.4)\cos(9.46) = 11.21\ m/s$$

Step 2 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(8.14)^2 + (11.21)^2} = 13.85\ m/s$$

Since the velocity of the rock, V_{CHECK} (= 13.85 m/s) is greater than the minimum velocity, V_{MIN} (= 1.0 m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step.

Step 3

In a similar fashion to the previous step, the final rock conditions for step 2 are used as the initial conditions for step 3. The necessary parameters are determined and the quadratic equation is solved to find the time to intersection with the sixth slope segment:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(0-0)}{(89-46)} = 0$$

$$a = \frac{1}{2} g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = 11.21 - (0)8.12 \cong 11.21$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 21.9 - 0 + 0(46 - 26.8) \cong 21.9$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(11.21) \pm \sqrt{(11.21)^2 - 4(-4.90)(21.87)}}{2(-4.90)} \cong -1.229 \; or \; 3.54 \; s$$

t = -1.229 s is rejected because *t* must lie in the range [0,∞]. The intersection point and pre-impact velocity are determined:

$$X_I = V_{X0}t + X_0 = 8.12(3.54) + 26.8 = 55.642 \; m$$

$$Y_I = \frac{1}{2} gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(3.54)^2 + 11.21(3.54) + 21.9 = 0.000 \; m$$

$$V_{XB} = V_{X0} = 8.12 \; m/s$$

$$V_{YB} = V_{Y0} + gt = 11.21 + (-9.81)3.54 = -23.55 \; m/s$$

The velocities are transformed into components normal and tangential to the slope segment:

$$\theta = \tan^{-1}(q) = 0.0^0$$

$$V_{NB} = (V_{YB})\cos(\theta) - (V_{XB})\sin(\theta) = (-23.55)\cos(0) - (8.12)\sin(0) = -23.5 \; m/s$$

$$V_{TB} = (V_{YB})\sin(\theta) + (V_{XB})\cos(\theta) = (-23.55)\sin(0) + (8.12)\cos(0) = 8.14 \; m/s$$

The impact is calculated by multiplying by the coefficients of restitution:

$$V_{NA} = R_N V_{NB} = 0.4(-23.5) = -9.42 \; m/s$$

$$V_{TA} = R_T V_{TB} = 0.6(8.14) = 4.88 \; m/s$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = (V_{NA})\sin(\theta) + (V_{TA})\cos(\theta) = (-9.42)\sin(0) + (4.88)\cos(0) = 4.88 \; m/s$$

$$V_{YA} = (V_{TA})\sin(\theta) - (V_{NA})\cos(\theta) = (4.88)\sin(0) - (-9.42)\cos(0) = 9.42\ m/s$$

Step 3 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(4.88)^2 + (9.42)^2} = 10.6\ m/s$$

Since the velocity of the rock, V_{CHECK} (=10.6 m/s) is greater than the minimum velocity, V_{MIN} (= 1.0 m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step.

Step 4

The final rock conditions for step 3 are used as the initial conditions for step 4. The necessary parameters are determined and the quadratic equation is solved to find the time to intersection with the sixth slope segment:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(0-0)}{(89-46)} = 0$$

$$a = \frac{1}{2} g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = 9.42 - (0)4.88 \cong 9.42$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 0 - 0 + 0(46 - 55.6) = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(9.42) \pm \sqrt{(9.42)^2 - 4(-4.90)(0)}}{2(-4.90)} \cong 0\ or\ 1.921\ s$$

t = 0 s is rejected because this is the starting point of the trajectory (we are already at *that* root). The intersection point and pre-impact velocity are determined:

$$X_I = V_{X0}t + X_0 = 4.88(1.921) + 55.6 = 65.021\ m$$

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(1.921)^2 + 9.42(1.921) + 0 = 0.000\ m$$

$$V_{XB} = V_{X0} = 4.88\ m/s$$

$$V_{YB} = V_{Y0} + gt = 9.42 + (-9.81)1.921 = -9.42\ m/s$$

The velocities are transformed into components normal and tangential to the slope segment:

$$\theta = \tan^{-1}(q) = 0.0^0$$

$$V_{NB} = (V_{YB})\cos(\theta) - (V_{XB})\sin(\theta) = (-9.42)\cos(0) - (4.88)\sin(0) = -9.42\ m/s$$

$$V_{TB} = (V_{YB})\sin(\theta) + (V_{XB})\cos(\theta) = (-9.42)\sin(0) + (4.88)\cos(0) = 4.88\ m/s$$

The impact is calculated by multiplying by the coefficients of restitution:

$$V_{NA} = R_N V_{NB} = 0.4(-9.42) = -3.77\ m/s$$

$$V_{TA} = R_T V_{TB} = 0.6(4.88) = 2.93\ m/s$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = (V_{NA})\sin(\theta) + (V_{TA})\cos(\theta) = (-9.42)\sin(0) + (2.93)\cos(0) = 2.93\ m/s$$

$$V_{YA} = (V_{TA})\sin(\theta) - (V_{NA})\cos(\theta) = (2.93)\sin(0) - (-9.42)\cos(0) = 3.77\ m/s$$

Step 4 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(2.93)^2 + (3.77)^2} = 4.77\ m/s$$

Since the velocity of the rock, V_{CHECK} (= 4.77 m/s) is greater than the minimum velocity, V_{MIN} (= 1.0 m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step. However, the hand calculations will not continue because they are very similar to step 4, and will not provide much further verification, only repetition.

Conclusion

The same geometry and parameters were input into RocFall and a simulation was performed. The results from RocFall were compared to the manual calculations. The results from the manual calculations were identical to the RocFall results for all practical purposes. The impact locations calculated by hand agreed with the program results up to the third decimal place in all cases (i.e. less than 0.5 mm difference, everywhere). Therefore, the projectile algorithm seems to be working correctly.

Although co-ordinate output was not available from the comparison program, the graphical output matches that of RocFall (as can be seen by comparing Figures A.1.1 and A.1.2). This correlation is a good indication that the programs are performing the calculations as desired.

Since the comparison program produces results that are very similar to the results produced by RocFall, and the theoretical basis (the equations used) for the two programs are the same, it is reasonable to conclude that both programs are working correctly. The comparison of the results produced by these two programs does not prove the validity of the equations; however, it does provide greater confidence that the equations were properly coded into the programs.